LETTER TO THE EDITOR

Comment on Electrostatics of Membranes

Dear Sir:

Adams (1972) has proposed that a curved membrane consisting of radially oriented dipoles experiences an outward pressure due to attractive forces between the two layers of charge which have slightly different charge densities. The same result can be obtained by considering the repulsive forces within each layer of charge. This is equivalent to viewing the membrane as a capacitor, and realizing that, keeping the thickness constant, a decrease in potential energy would be associated with an increase in area. A pressure can be associated with this tendency to expand.

Consider a capacitor consisting of two oppositely charged, concentric, conducting shells, one at a radius of R_1, the other at a radius of R_2 where $R_2 > R_1$. Let ϵ be the permittivity between the shells and Q be the charge on each. The capacitance of this arrangement is given by

$$C = \frac{4\pi\epsilon R_1 R_2}{R_2 - R_1}. \qquad (1)$$

The potential energy of a capacitor is

$$W = \frac{Q^2}{2C}, \qquad (2)$$

which, in this case, becomes

$$W = \frac{Q^2(R_2 - R_1)}{8\pi\epsilon R_1 R_2}. \qquad (3)$$

The change in this energy, with respect to a change in radius, can be written as

$$\mathrm{d}W = \frac{Q^2}{8\pi\epsilon}\left(\frac{-\mathrm{d}R_1}{R_1^2} + \frac{\mathrm{d}R_2}{R_2^2}\right), \qquad (4)$$

which, if $R_1 \cong R_2 = R$, becomes

$$\mathrm{d}W = \frac{Q^2(R_1^2 - R_2^2)}{8\pi\epsilon R^4} \cdot \mathrm{d}R. \qquad (5)$$

One also can write, however, $\mathrm{d}W = -P\,\mathrm{d}V$ or, in terms of R,

$$\mathrm{d}W = -P(4\pi R^2)\,\mathrm{d}R. \qquad (6)$$

Comparing equations 4 and 6 leads to

$$P = \frac{1}{4\pi R^2} \cdot \frac{Q^2(R_2^2 - R_1^2)}{8\pi\epsilon R^4}; \tag{7}$$

Substituting $Q = VC$, where V is the membrane potential and C is as given in equation 1, yields

$$P = \frac{(R_2^2 - R_1^2)V^2\epsilon}{2R^2(R_2 - R_1)^2}. \tag{8}$$

Then, if $R_1 + R_2 \cong 2R$ and $R_2 - R_1 = t$, one obtains

$$P = \frac{\epsilon V^2}{Rt} \tag{9}$$

which is the same as Adams's equation 24.

The result is a consequence of electrostatic charges. Interpreting it in terms of attraction between opposite charges is hardly more appealing than an interpretation in terms of repulsion between like charges.

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REFERENCE

ADAMS, K. H. 1972. *Biophys. J.* **12**:123.

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